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LETTER TO THE EDITOR

Scaling of rough surfaces in a (2+1)-dimensional growth model with nonlinear anisotropic diffusion

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Abstract. We present surface properties of a nonlinear random deposition model taking into account anisotropy in diffusion, which appears in some experimental situations. We report the results of simulations made in a bidimensional substrate. We find that the surface width $w(t, L)$ scales as t^β with $\beta = 0.35$ for small t and the saturation value does not depend on L. A crossover phenomenon associated with large length scales is studied.

Aggregation and growth take place in a wide variety of physical, chemical and biological processes, including solidification, vapour deposition, fluid flow in porous media and growth of bacterial colonies. Recently, considerable attention has been given to the characterization of the surface structure during growth processes due to the practical interest and importance in scientific and industrial applications.

Thin films are commonly grown through molecular beam epitaxy (MBE). Incoming atoms move ballistically with very long mean free paths in a direction normal to the substrate, until they are deposited on the surface. The growth rate of the films is determined by their local environment and the roughness characterizing the surface.

The properties of rough surfaces have been studied using different computational models. The simplest one, which resembles MBE, is the random deposition model. Particles simply 'rain' down onto a substrate, moving along straight line trajectories until they reach the top of the column in which they were dropped, at which point they stick to the deposit and become part of the aggregate. Fluctuations in the column heights obey a Poisson process because there is no correlation between columns. A modified random deposition model that accounts for the finite surface diffusion which exist in most realistic situations was considered later [1]. The effect of the introduction of surface diffusion is the appearance of non-trivial correlations between different columns, and it has been shown [1] that the surface of the deposit exhibits a self-affine fractal geometry that can be described in terms of the scaling form

$$w(L, h) = L^\alpha f(h/L^z) \tag{1}$$

where w is the variance of the surface height, h is the mean height of the deposit, and L is the lateral size of the deposit. The scaling function $f(x)$ tend to a constant value when x approaches infinity and behaves as x^β when x approaches zero, with $\beta = \alpha/z$. $w(L, h)$ behaves as L^α for large h and as h^β for large L.

In 1+1 dimensions, with constant diffusion coefficient, numerical simulations [1] show that the exponents α and z approach to the theoretical values predicted by

Edwards and Wilkinson [2] by solving a linear Langevin equation for the growing surface. The predicted values in $d+1$ dimensions are $\alpha=(2-d)/2$, $\beta=(2-d)/4$ and $z=2$. For $d=1$ these values are $\frac{1}{2}$, $\frac{1}{4}$ and 2 respectively. The ballistic model and the Eden model also follow the scaling function (1), and simulations [3] give α equal to $\frac{1}{2}$ and β about $\frac{1}{3}$. The same value of the β exponent is obtained when complete restructuring is included in the ballistic deposition model.

A variation on the Edwards and Wilkinson model which includes the nonlinear effect of a driving force was introduced by Kardar, Parisi and Zhang (KPZ) [4]. The KPZ equation represents a universality class for interface dynamics far from equilibrium and predicts the exact relation $\alpha+z=2$ with $\alpha=\frac{1}{2}$ and $z=\frac{3}{2}$, with $z=\alpha/\beta$.

The ability of the incoming particle to diffuse, however, depends not only on the existence of local minima in the height of the surface but also on the local slope of the aggregate [2]. This dependence is given by the angle of repose for a granular material. The tangent of the angle of repose is the steepest slope the material can sustain. In order to model the influence of the local slope of the aggregate at the minima a threshold for the diffusivity is introduced, and thus a random deposition model with nonlinear diffusion is obtained.

The scaling properties of $(1+1)$-dimensional models with different types of nonlinear diffusivity has been recently studied [5-7]. Results of a numerical simulation of a $(1+1)$-dimensional model with constant threshold are reported in reference [5]. The exponents α and β remain equal to those for linear diffusion although the zone at which the roughness scales with h as well as the saturation value are different and depend on the threshold. In reference [6] the nonlinearity is introduced as a dependence of the diffusivity on the height of the deposit h and $D\propto h^k$, with positive and negative values of k. Later, numerical simulations have confirmed this result, extending the study to the range where $w(L, h)$ reaches the saturation value [7].

In a $(d+1)$-dimensional simulation of random deposition the substrate has L^d columns into which particles are dropped. The column in which the particle falls is chosen randomly and different deposition models are defined depending on how and where the particle sticks on the deposit. The deposited particle can diffuse on the surface until it finds the column with the minimum height within a finite distance from the column in which it was dropped.

We study the behaviour of the surface roughness of a deposition model with nonlinear anisotropic diffusion on lattices in which particles are deposited from above onto a bidimensional substrate of sites with periodic boundary conditions. Anisotropy in diffusion has been reported in experimental aggregation processes, for example on Si(001) 2×1 surfaces [8], but has not been yet considered in simulation of deposition models. From a physical base, anisotropy in diffusion is due to the ability of the deposited particle to diffuse in one direction more easily than in the other.

We carried out our simulation as follows: particles are allowed to fall vertically down until they reach the substrate or another particle in the deposit. A particle, randomly dropped in column (i,j) will be allowed to diffuse to the smallest of the nearest-neighbour columns if the height difference Z is greater than a critical value Z_c. The diffusion is nonlinear due to the existence of the threshold Z_c. In order to introduce the anisotropy in the diffusivity, two threshold values were defined, Z_cx and Z_cy.

Results of the simulation for isotropic diffusion with $Z_cx=Z_cy=0$ and $Z_cx=Z_cy=5$ are shown in figure 1, for substrate dimensions 20×20. It can be seen that $\alpha=0$ because the roughness of the surface does not scale with the size of the substrate, and

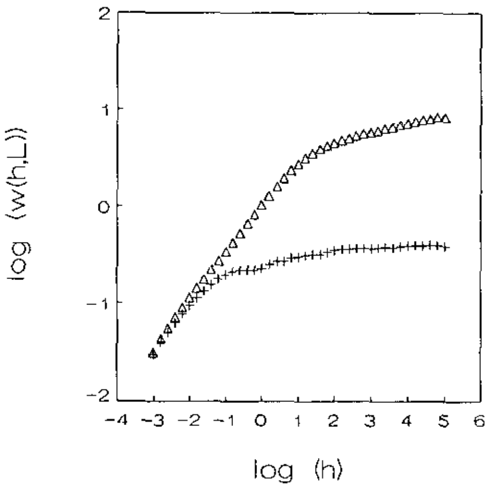

Figure 1. Results of the simulation for isotropic diffusion with $Z_c x = Z_c y = 0$ (+) and $Z_c x = Z_c y = 5$ (Δ), for substrate dimensions 20×20.

$\beta = 0$ because the roughness reaches saturation immediately after leaving the zone of slope $\frac{1}{2}$. These results agree with previous simulations results [1] and with theoretical predicted values for the α and β exponents in $2+1$ dimensions [2].

Results for anisotropic diffusion, i.e. $Z_c y = 0$ and $Z_c x \neq 0$ are shown in figure 2 for substrate dimensions 10×10, 20×20 and 40×40. Comparing with figure 1, it can be seen that a scaling zone with slope less than $\frac{1}{2}$ appears before the saturation is reached. The exponent β, equal to the slope of this zone, is $\beta = 0.356 \pm 0.002$ and it was found that it does not depend on the size of the substrate, since the same value of β is obtained for different studied lengths L. The exponent α is zero, because the saturation value of the roughness does not depend on the lateral size of the substrate L as can be seen in figure 2.

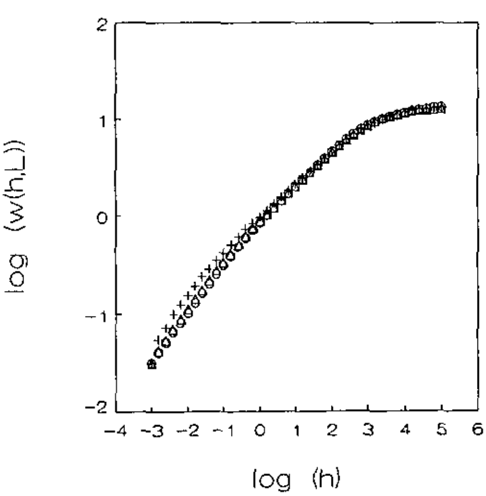

Figure 2. Results of the simulation for anisotropic diffusion with $Z_c y = 0$ and $Z_c x = 10$, for substrate dimensions 10×10 (+), 20×20 (Δ) and 40×40 ($\bigcirc$).

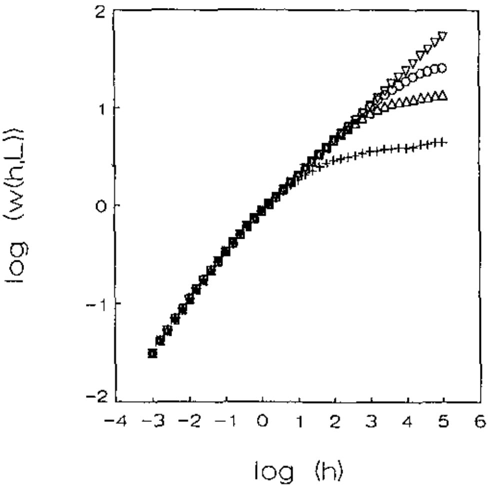

Figure 3. Results of the simulation for different degrees of anisotropy for substrate dimensions 20×20. (+): $Z_c x = 5$, $Z_c y = 0$; (Δ): $Z_c x = 10$, $Z_c y = 0$; ($\bigcirc$): $Z_c x = 15$, $Z_c y = 0$; (∇): $Z_c x = 50$, $Z_c y = 0$.

We also studied the dependence of β on the degree of anisotropy. The results are displayed in figure 3 for $Z_c x = 5$, $Z_c x = 15$ and $Z_c x = 50$. The exponent is β independent of $Z_c x$ since the same slope is obtained for different anisotropies. However, the saturation value of the roughness depends on the magnitude of anisotropy.

The surface width, then, behaves as h^β for $h_c < h < h_s$, as $h^{1/2}$ for $h < h_c$, and does not scale with L for $h > h_s$. As can be seen in figure 3, h_c does not depend on the degree of anisotropy. This indicates that the crossover phenomenon associated with h_c reported for $1+1$ dimensions [9] does not appear in $2+1$ dimensions. Instead, we analysed a crossover phenomenon associated with h_s. For $h \gg L$ the slope of the roughness changes with $Z_c x$ from 0 to β as $Z_c x$ increases. The values of h_s, from figure 3, versus $Z_c x$ are plotted in figure 4, and it can be seen that h_s scales as $h_s \sim (1/Z_c x)^{-1/\phi}$, where $\phi = 0.59$. This value is close to the slope reported in [9] for the crossover length h_c in $1+1$ dimensions.

For the KPZ continuous model $d = 2$ (in $d+1$ dimensions) is a critical dimension in which $\beta = 1/3$ and $\alpha = 1/2$. The same values for α and β are obtained for the Eden model [3] and were argued to be *superuniversal* (independent of d) [4]. Comparing the exponent β and α for anisotropic diffusion in $d = 2$ we note that the first one is close to the KPZ and Eden β, although our simulations indicate that $\alpha = 0$.

In conclusion, we have studied a surface growth model with nonlinear diffusion in a $(2+1)$-dimensional lattice, which represents experimental aggregation processes. We found that for isotropic diffusion, the width of the surface scales as the Edwards and Wilkinson model predicts, i.e. the exponents α and β of the scaling function are zero. If anisotropy in diffusivity is taken into account, non-trivial scaling with the surface height is obtained. A β exponent equal to 0.35 appears and this exponent depends neither on the amount of anisotropy nor on the size of the substrate. The α exponent remains zero. A crossover phenomenon associated with large length scales was found. Although the phenomenon is different from that in $1+1$ dimensions, the crossover length scales with a similar exponent.

To our knowledge, continuous models for anisotropic diffusion are not available at present, so the results reported here must wait for comparison.

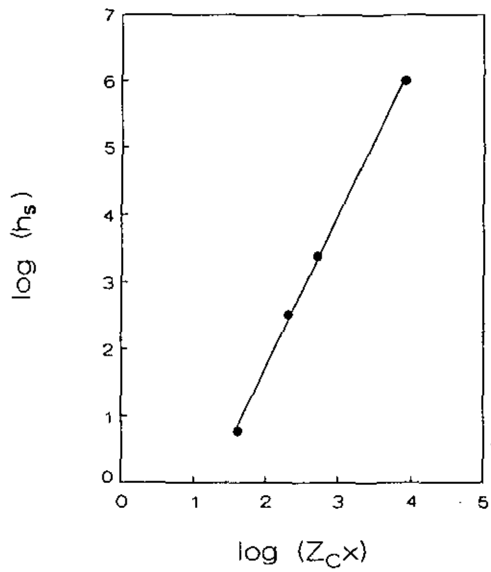

Figure 4. The log-log plot of h_s versus $Z_c x$. The slope of the straight line through the points is $1/\phi = 1.69$.

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References

[1] Family F 1986 *J. Phys. A: Math. Gen.* **19** L441
[2] Edwards S and Wilkinson D 1982 *Proc. R. Soc.* A **381** 17
[3] Family F 1990 *Physica* A **168** 561
[4] Kadar M, Parisi G and Zhang Y 1986 *Phys. Rev. Lett.* **56** 9
[5] Arizmendi C and Sanchez J 1991 *Proc. XV Int. Workshop on Condensed Matter Theories* ed A Proto and J Aliaga (New York: Plenum) in press
[6] Nagatani T 1991 *Phys. Rev.* A **43** 10
[7] Arizmendi C and Sanchez J 1992 *J. Phys. A: Math. Gen.* submitted
[8] Wang J and Rocket A 1991 *Phys. Rev.* B **43** 15
[9] Nagatani T 1990 *J. Phys. A: Math. Gen.* **23** L1139